Partition function $R(n)$ to give ideas for Goldbach's conjecture

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PARTITION FUNCTION R(N) TO GIVE IDEAS FOR GOLDBACH’S CONJECTURE

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ABSTRACT. In this article, we introduce a new kind of partition that applies to solve problems in number theory. As we know, Goldbach’s conjecture has not been solved so far; this new partition function R(n) has been established to facilitate and give ideas by providing strong hypothesis. Finally, we give another proposition for a particular case of even natural number.

1. INTRODUCTION

1.1 Definition of Partition

A partition of a positive integer n, also known as an integer partition, is a way to express n as the sum of positive integers in number theory. Two sums that differ only in the order of their summands are considered the same partition. (For ex: 2+1 and 1+2)

P(n): The number of partitions of n, also called the partition function. In number theory, the partition function P(n) represents the number of possible partitions of a natural number n, which is to say the number of distinct ways of representing n as a sum of natural numbers (with order irrelevant). By convention we have P(0) = 1, P(n) = 0 for n negative. Indian great mathematician Srinivasa Ramanujan was perhaps the first mathematician who seriously investigate the properties of partition function P(n) [1].
For instance, there are five distinct ways to divide the number 4:

4
3+1
2+2
2+1+1
1+1+1+1

The partition function \( p(n) \) determines how many partitions there are for \( n \). So, \( p(4) = 5 \). A partition of \( n \) is indicated by the notation \( n \).

\( \text{P}(120052058) \), which has 12,198 decimal digits, is the highest known prime number that counts a number of partitions as of June 2013. In 1917, Hardy and Ramanujan jointly presented an amazing asymptotic formula as follows:

\[
P(n) \approx \frac{1}{4\sqrt{3n}} \exp \left( \frac{\pi \sqrt{2n/3}}{3} \right) \text{ as } n \to \infty.
\]

1.2 Description of the new partition

Let \( R(n) \) be the number of possibilities to write a number \( n \) as the sum of two natural numbers i.e \( R(n)=a+b=c+d= \ldots \)

For \( n=0 \), \( R(0) = 0+0 \), only one possibility

\( n=1 \), \( R(1) = 1+0 \), the same number of possibilities as 0

\( n=2 \), \( R(2) = 2+0 = 1+1 \), two possibilities

\( n=3 \), \( R(3) = 3+0 = 2+1 \), the same number of possibilities as 2

We saw that:

\[
R(0) = R(1) = 1
\]
\[
R(2) = R(3) = 2
\]

\[
\ldots
\]
\[
\ldots
\]

\[
R(2n) = R(2n+1) \text{ for all natural numbers } n
\]
Let’s continue, and let the number of the possibilities in the right side

\[
\begin{align*}
R(4) &= 4+0 = 3+1 = 2+2 \quad 3 \text{ possibilities} \\
R(5) &= 5+0 = 4+1 = 3+2 \quad 3 \text{ possibilities} \\
R(6) &= 6+0 = 5+1 = 4+2 = 3+3 \quad 4 \text{ possibilities} \\
R(7) &= 7+0 = 6+1 = 5+2 = 4+3 \quad 4 \text{ possibilities} \\
R(8) &= 8+0 = 7+1 = 6+2 = 5+3 = 4+4 \quad 5 \text{ possibilities} \\
R(9) &= 9+0 = 8+1 = 7+2 = 6+3 = 5+4 \quad 5 \text{ possibilities} \\
R(10) &= 10+0 = 9+1 = 8+2 = 7+3 = 6+4 = 5+5 \quad 6 \text{ possibilities} \\
R(11) &= 11+0 = 10+1 = 9+2 = 8+3 = 7+4 = 6+5 \quad 6 \text{ possibilities} \\
R(12) &= 12+0 = 11+1 = 10+2 = 9+3 = 8+4 = 7+5 = 6+6 \quad 7 \text{ possibilities} \\
R(13) &= 13+0 = 12+1 = 11+2 = 10+3 = 9+4 = 8+5 = 7+6 \quad 7 \text{ possibilities} \\
R(14) &= 14+0 = 13+1 = 12+2 = 11+3 = 10+4 = 9+5 = 8+6 = 7+7 \quad 8 \text{ possibilities} \\
R(15) &= 15+0 = 14+1 = 13+2 = 12+3 = 11+4 = 10+5 = 9+6 = 8+7 \quad 8 \text{ possibilities}
\end{align*}
\]

As we can see, that is an arithmetical progression with common ratio 1. Thus, we are able to write \( R(2n) = R(2n-2) + 1 \) for all positive integers \( n \). That gives us the idea of considering just the \( R(n) \) for \( n=2k, k \in \mathbb{N} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R(n) )</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 1.

This leads us to know the number of possibilities that a number can be written as the sum of two numbers \( R(n) = X \)

We can find \( X \) for all postive integers \( n \).

Let’s see!

\[ \begin{align*}
R(0) &= 1 \\
R(2) &= 2 \\
R(4) &= 3 \\
R(6) &= 4
\end{align*} \]

What is the relation between \( X \) and \( n \) based on those 4?

\[ \begin{align*}
0 &= 2(1) - 2 \text{ for } n=0 \\
2 &= 2(2) - 2 \text{ for } n=2 \\
4 &= 2(3) - 2 \text{ for } n=4 \\
6 &= 2(4) - 2 \text{ for } n=6
\end{align*} \]
It leads us to think that \( n=2X-2 \Rightarrow X = \frac{n+2}{2} \) (for all integers \( n \)) because the number of possibilities is the same for \( n \) and \( n+1 \) for all even natural numbers \( n \).

So, the formula to find \( R(n) \) is:

\[
R(n) = \frac{n+2}{2}
\]

for all even natural numbers \( n \).

**Proof**

For \( n=0 \),

\[
R(0) = \frac{0+2}{2} = 1, \text{ true}
\]

Let \( R(n) \) be true, i.e, \( R(n) = \frac{n+2}{2} \) for all even numbers \( n \).

And then, let’s show that it’s also true for \( n+2 \).

\[
R(n+2) = \frac{(n+2)+2}{2}, \text{ it’s also true because } R(n+2) \text{ can be written as } R(n+2) = \frac{n’+2}{2} \text{ with } n’=n+2
\]

By mathematical induction, we can conclude that:

\[
R(n) = \frac{n+2}{2}, \text{ for all even natural numbers } n
\]

**Results**

\( R(n) = \frac{n+2}{2} \) For all even numbers \( n \)

\( R(2n) = R(2n-2) + 1 \)

\( R(n) = n = \Rightarrow n=2 \)

2. APPLICATION OF THE PARTITION FUNCTION ON THE GOLDBACH’S CONJECTURE

Now let’s look at the relationship between what we discussed above and Goldbach's unresolved conjecture.
2.1 Discussion of the Goldbach’s Conjecture

Goldbach’s conjecture is an unsolved mathematical problem within number theory that was formulated by the German mathematician Christian Goldbach in letter correspondence with the famous Swiss mathematician Leonhard Euler in the year 1742. The problem sounds fairly simple in its statement but yet no one has achieved a solution for the original problem, and it still draws the attention of mathematicians even to this date, more than 250 years after it was proposed. Worth mentioning is that the weak form of the conjecture, the ternary Goldbach’s conjecture, was claimed to have been solved in 2014 by the Peruvian mathematician Harald Helfgott [2].

Definition 2.1.1: Every even number bigger than 2 can be expressed as the sum of two primes (binary or strong Goldbach’s conjecture).

Definition 2.1.2: (Goldbach partition [3]) The pair of two prime numbers, p and q, where p + q = n and n being an even integer, is called a Goldbach partition of n.

Several examples of partitions of the binary conjecture are given in the table below.

<table>
<thead>
<tr>
<th>Even integer:</th>
<th>10</th>
<th>22</th>
<th>48</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partitions</td>
<td>3+7</td>
<td>3+19</td>
<td>5+43</td>
<td>11+139</td>
</tr>
<tr>
<td></td>
<td>5+5</td>
<td>5+17</td>
<td>7+41</td>
<td>13+137</td>
</tr>
<tr>
<td></td>
<td>11+11</td>
<td>11+37</td>
<td>19+131</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>17+31</td>
<td>23+127</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>19+29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of partitions:</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>15</td>
</tr>
</tbody>
</table>

Table 2.

As we know, Golbach’s conjecture states that every even integer is the sum of two prime numbers. Therefore, we can consider the relationship between it and the one that we discussed above because we have seen how a number can be written as a sum of two natural numbers.
2.2 Importance of the partition to the conjecture

\[ R(n) = X \]

- If \( X = 2k \)
  
  Then we must have \( h \) sums of even numbers and \( h \) sums of odd numbers

  Eg: \( R(6) = 4 \). Here \( h = 2 \), 2 sums of odd numbers, the same for even numbers \((6+0, 4+2)\) and \((5+1, 3+3)\)

- If \( X = 2k+1 \)
  
  Then we have \( s \) sums of even numbers and \( s-1 \) sums of odd numbers with \( X = s + (s-1) \)

  Eg: \( R(12) = 7 \)

  Here \( s = 4 \), \( s-1 = 3 \)

  4 sums of even number and 3 sums of odd number

  \((12+0, 10+2, 8+4, 6+6)\) and \((11+1, 9+3, 7+5)\)

Let’s show it on a table.

<table>
<thead>
<tr>
<th>( n )</th>
<th>Sums of even numbers</th>
<th>Sums of odd numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n=4 ) ( R(4) = 3 )</td>
<td>4+0 2+2</td>
<td>3+1</td>
</tr>
<tr>
<td>( n=6 ) ( R(6) = 4 )</td>
<td>6+0 4+2</td>
<td>3+3 5+1</td>
</tr>
<tr>
<td>( n=8 ) ( R(8) = 5 )</td>
<td>8+0 4+4 6+2</td>
<td>7+1 5+3</td>
</tr>
<tr>
<td>( n=10 ) ( R(10) = 6 )</td>
<td>10+0 8+2 6+4</td>
<td>9+1 7+3 5+5</td>
</tr>
<tr>
<td>( n=14 ) ( R(14) = 8 )</td>
<td>14+0, 12+2 10+4 8+6</td>
<td>13+1, 7+7 11+3 9+5</td>
</tr>
</tbody>
</table>

Table 3.
Let us concentrate on the sum of odd numbers only, so that we can delete the whole sum of even numbers. We will paint them red. And the whole first case of odd numbers, because one (1) is not a prime number, so we don’t need them. We will paint them yellow.

\[ R(4) = 4+0 = 3+1 = 2+2 \], special case because 2 is prime
\[ R(6) = 6+0 = 5+1 = 4+2 = 3+3 \]
\[ R(8) = 8+0 = 7+1 = 6+2 = 5+3 = 4+4 \]
\[ R(10) = 10+0 = 9+1 = 8+2 = 7+3 = 6+4 = 5+5 \]
\[ R(12) = 12+0 = 11+1 = 10+2 = 9+3 = 8+4 = 7+5 = 6+6 \]
\[ R(14) = 14+0 = 13+1 = 12+2 = 11+3 = 10+4 = 9+5 = 8+6 = 7+7 \]
\[ R(16) = 16+0 = 15+1 = 14+2 = 13+3 = 12+4 = 11+5 = 10+6 = 9+7 = 8+8 \]
\[ R(18) = 18+0 = 17+1 = 16+2 = 15+3 = 14+4 = 13+5 = 12+6 = 11+7 = 10+8 = 9+9 \]
\[ R(20) = 20+0 = 19+1 = 18+2 = 17+3 = 16+4 = 15+5 = 14+6 = 13+7 = 12+8 = 11+9 = 10+10 \]
\[ R(22) = 22+0 = 21+1 = 20+2 = 19+3 = 18+4 = 17+5 = 16+6 = 15+7 = 14+8 = 13+9 = 12+10 \]
\[ = 11+11 \]

**Interpretation:**

Hopefully, among these two-dimensional additions, there is at least one sum of two primes because it is known as the number increases, the possibility of writing it as a combination of two natural numbers increases. And if there are at least three ways of writing a number as a combination of two odds number, there must be two prime numbers among these three ways.

For example,

\[ R(22) = 19+3 = 17+5 = 15+7 = 13+9 = 11+11 \]

Here, we have 6 prime numbers which are 3, 5, 7, 11, 13, 17, 19.
3. **HYPOTHESIS**

When I was trying to come up with an answer for this conjecture, I was thinking that studying the even numbers at each interval can be beneficial to us so that we can figure out any changes. Basically, I want to find out the ability of the even number to be written as the sum of two prime numbers. The following pictures show my experiments:

The graph illustrates the even numbers along the x-axis and the possibility of writing them as sum of two prime numbers along the y-axis.

**FIG 1.**

**FIG 2.**
Figure 1. shows the range of possibilities from 0 to 10,000. Due to the presence
of 2, we have 0 possibilities. However, if we start with 4, the lowest possibility
is 1, and as the number rises, the likelihood of writing it as the product of two
prime integers rises.

Figure 2. illustrates the variation between 10,000 and 20,000. As we can see, the
minimum possibility of writing an even number as the sum of two prime
numbers increases from what we have seen previously. Same interpretation for
Fig 3. (20,000 to 30,000), Fig 4. (30,000 to 40,000) and Fig 5. (50,000 to
60,000). The smallest number of possibilities in this last range is 328.

3.1 Interpretation

The minimum possibility of representing an even integer as the sum of two
primes for each interval of even numbers [a, n] where a is an even number
higher or equal to 4 is x such that x is strictly greater or equal to 1.

4.PROPOSITION

First of all, we should know what twin prime numbers are?

4.1 Definition: A twin prime is a prime number that is either 2 less or 2 more
than another prime number—for example, either member of the twin prime pair
(17, 19) or (41, 43). In other words, a twin prime is a prime that has a prime
gap of two. Sometimes the term twin prime is used for a pair of twin primes; an
alternative name for this is prime twin or prime pair. [4]

4.2 Theorem: There are infinitely many twins prime numbers [5].
Following are examples of twin prime numbers (3,5), (5,7), (11,13), (17, 19)
(29,31) (59,61) (71,73) ………

4.3 Proposition: Let n be an even natural number greater than 2. If n can be
written as the sum of two twin prime numbers, then the next even number has to
be a sum of two prime numbers.

4.4 Proof:
Suppose n=p+q with p-q=2, where n is even number greater than two and (p,q)
are two prime numbers.
Then n+2 = p+q+2, as p-q=2 => q=p-2
= p+(p-2) + 2
n+2 = p+p, true …
5. CONCLUSION

This article introduces a new approach on Goldbach’s conjecture and advances our understanding on it. If all possible ways to turn a number into the sum of two numbers are known, it is easier and faster to estimate if this conjecture is really true. Moreover, we know that it is always true up to $4.10^{18}$, but it’s good to examine the changes in each range to see how it behaves.

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